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The Army has in recent years refined its methods of procuring aircraft. Until recently the Army had only procured aircraft which were more or less off-the-shelf items. That is, the aircraft were either developed for other military services or for civilian use and were adapted to meet Army needs. As a result, the Army was not significantly involved in the writing of specifications which affected the overall design of the vehicle. Operational experience has indicated, however, the need for specifications which reflect the particular requirements of Army aviation. This paper is concerned with requirements which have been imposed in the area of helicopter mechanical instability, or ground resonance as this phenomenon is commonly known, and the impact which these requirements have on the analyst.

The ground resonance problem has been with rotary wing aircraft since their inception, but the phenomenon was not well understood until many years after its first occurrence on the early autogyros. The instability was originally thought to be aeromechanical in nature, however, later analysis confirmed that the instability could be predicted based solely on mechanical considerations. The report by Coleman and Feingold (1) has become the classical reference on this problem. This classical treatment of the problem was expanded by Brooks (2) and Bielawa (3) to include additional degrees of freedom and to experimentally verify the analysis.

All these analyses showed that the ground resonance instability involved a mechanical coupling of the inplane degrees of freedom of the rotor blades with the rigid body degrees of freedom of the helicopter on its undercarriage. The analytical results also indicated that the instability could be eliminated within the operating

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rotor speed range of any particular helicopter by properly selecting (i) the damping and stiffness characteristics of the undercarriage and (ii) the inplane damping of the blades.

Because of operational considerations, the Army has imposed requirements that the helicopter not only be shown free from mechanical instability under normal operating conditions but freedom from instability must also be shown for a variety of abnormal conditions. Some of these abnormal conditions are: operation on ice, flat tire and flat strut on one side, operation from a 12° slope on any heading, and operation with one blade damper inoperative. All but the last of these requirements can be handled by appropriate modifications of the basic parameters in the classical analysis of Reference 1. As will be shown later, the requirement for stability with one blade damper inoperative causes one of the basic assumptions in the classical analysis to be violated and thus a new analysis method must be developed.

The investigation of the ground resonance problem when one blade damper is inoperative is the subject of this paper. A method will be presented for handling the problem and comparison will be made with two methods which have previously been used in attempting to satisfy the Army requirement. Instabilities which arise when one blade damper is inoperative will be examined and, finally, means for eliminating these instabilities will be discussed.

SYMBOLS

c_i	Lag damping rate
c_x	Effective hub damping in x-direction (longitudinal)
c_y	Effective hub damping in y-direction (lateral)
e	Lag hinge offset
I_b	Second mass moment of blade about lag hinge
k_i	Lag spring rate
k_x	Effective hub stiffness in x-direction (longitudinal)
k_y	Effective hub stiffness in y-direction (lateral)
m_b	Blade mass

m_x	Effective hub mass in x-direction
m_y	Effective hub mass in y-direction
N	Number of blades in rotor
S_b	First mass moment of blade about lag hinge
t	Time
x_h, y_h	Coordinates of hub in fixed reference frame
ζ_i	Lag deflection of i^{th} blade
η_i	Defined by Equations (2)
ν_o	Defined by Equations (2)
ψ_i	Azimuthal location of i^{th} blade
Ω	Rotor speed
ω_{o_i}	Defined by Equations (2)

MATHEMATICAL FORMULATION

A complete derivation of the equations of motion for the ground resonance problem is presented in Reference 4. This development will not be repeated here, but the equations together with the underlying assumptions will be given in order to discuss the implications of the one-blade-damper-inoperative requirement.

It is assumed, as is done in Reference 1, that the helicopter on its landing gear can be represented by effective parameters applied at the rotor hub. It is further assumed that only inplane motions of the hub and blades are important in determining the ground resonance characteristics of the helicopter. Thus the degrees of freedom to be considered consist of two inplane hub degrees of freedom and one lead-lag degree of freedom for each blade in the rotor. The mathematical model to be used in the analysis is shown in Figure 1. Note that in the figure only a typical blade is shown. The analysis is formulated for a rotor having N blades, and each blade is assumed to have a rotational spring and damper which act about the lag hinge. Further, it is assumed that each of the blades may have different lag spring and lag damper characteristics. This last assumption is

necessary in order to be able to treat the one-blade-damper-inoperative situation and represents a major departure from the classical analysis. In References 1, 2, and 3 each blade was assumed to have identical properties and, as will be indicated later, this assumption leads to considerable simplification of the equations of motion. With respect to the hub degrees of freedom, it is assumed that, in the absence of the rotor, the longitudinal and lateral motions of the hub are uncoupled. This is an approximation, but it is an assumption made in Reference 1 and one generally used in mechanical stability analyses.

Based on the above assumptions, the equations of motion for the rotor-hub system may be written as

$$\left.\begin{aligned}
&\ddot{\zeta}_i + \eta_i \dot{\zeta}_i + \left(\omega_{o_i}^2 + \Omega^2 \nu_o^2\right) \zeta_i = \left(\nu_o^2/e\right)\left[\ddot{x}_h \sin \psi_i - \ddot{y}_h \cos \psi_i\right] \\
&\qquad\qquad i = 1, 2, \ldots, N \\
&\left(m_x + Nm_b\right) \ddot{x}_h + c_x \dot{x}_h + k_x x_h = S_b \sum_{i=1}^{N}\left[\left(\ddot{\zeta}_i - \Omega^2 \zeta_i\right) \sin \psi_i + 2\Omega \dot{\zeta}_i \cos \psi_i\right] \\
&\left(m_y + Nm_b\right) \ddot{y}_h + c_y \dot{y}_h + k_y y_h = -S_b \sum_{i=1}^{N}\left[\left(\ddot{\zeta}_i - \Omega^2 \zeta_i\right) \cos \psi_i - 2\Omega \dot{\zeta}_i \sin \psi_i\right]
\end{aligned}\right\} \quad (1)$$

where

$$\left.\begin{aligned}
\nu_o^2 &= eS_b/I_b \\
\omega_{o_i}^2 &= k_i/I_b \\
\eta_i &= c_i/I_b
\end{aligned}\right\} \quad (2)$$

and small displacement assumptions have been made on ζ_i, x_h, and y_h in order to linearize the equations. The parameter ψ_i describes the azimuthal position of the $i^{\underline{th}}$ blade at time t and is given by

$$\psi_i = \Omega t + 2\pi(i - 1)/N \tag{3}$$

The equations of motion for the system thus consist of (N + 2) coupled second-order differential equations in which the coupling terms have periodic coefficients. The periodic coefficients arise because the blade equations, the first N of Equations (1), are written in a rotating reference system whereas the hub equations, the last two of Equations (1), are written in a fixed system.

In order to eliminate the periodic coefficients in Equations (1) Coleman and Feingold (1) transformed the blade equations into the fixed reference system. This transformation, which greatly simplifies the equations of motion, is only possible if the rotor is isotropic with three or more blades. An alternate procedure is presented by Hammond (4) for eliminating the periodic coefficients if the rotor is nonisotropic, as is the case for one blade damper inoperative, but the hub is isotropic. This procedure involves transforming the hub equations of motion into the rotating frame of reference and requires that the rotor have two or more blades. Since the hubs of most, if not all, currently operational helicopters are nonisotropic this last procedure is only useful for determining the general nature of instabilities which occur when one blade damper is inoperative.

Thus for the general case of a nonisotropic rotor coupled with a nonisotropic hub one is faced with the problem of determining the stability of a system which is described by a set of second-order differential equations having periodic coefficients. It has been shown by Hammond (4) that the Floquet Transition Matrix method described by Peters and Hohenemser (5) and Hohenemser and Yin (6) provides an effective means for determining the stability characteristics of the system described by Equations (1). This method is essentially an eigenvalue method which is based on the Floquet-Liapunov theorem (7) for systems having periodic coefficients. Thus the stability characteristics of the system are direct outputs of the method.

Two methods used in the past for treating the one-blade-damper-inoperative ground resonance problem are (i) numerical integration of the equations of motion, and (ii) a smearing technique which involves a redistribution of damping over all the blades after one damper is considered imoperative. The reasoning for the second

approach is as follows: If the rotor has N blades then the total damping available in the rotor is Nc_i where c_i is the damping on one blade. If one damper is removed, the total damping becomes $(N - 1)c_i$. Thus, using the smearing approach, each blade in the rotor would be treated as if it had a lag damper rate equal to $c_i(N - 1)/N$. It is thus seen that this second approach analyzes a system which is quite different from the actual situation. The motivation behind this approach is to be able to use the standard Coleman and Feingold analysis for an isotropic rotor. In the results which follow, each of these techniques will be applied to a specific configuration and the ground resonance problem with one blade damper inoperative will be discussed in detail.

RESULTS

In order to illustrate the implications of one blade damper inoperative on the ground resonance characteristics of a single rotor helicopter, a set of parameters was chosen. These parameters were chosen so as to be in the general range of interest for single rotor helicopters and were such that the system was stable with all dampers functioning up to a rotor speed of 400 rpm. The parameters used in obtaining the results which follow are shown in Table 1.

Results for the system described by the parameters of Table 1 with all blade dampers operational are indicated in Figure 2. These results were obtained using the standard Coleman and Feingold approach. This approach results in only two equations which describe the rotor degrees of freedom regardless of the number of blades. Thus there are only four modes which result from the eigenvalue analysis. As can be seen from the upper portion of the figure, where the real parts of the eigenvalues are plotted as a function of rotor speed, the system is stable over the entire rotor speed range. The labeling on the various modes is intended for identification purposes only and is not meant to imply anything with respect to the character of the modes. In the lower portion of the figure is plotted the frequencies of the various modes as a function of rotor speed. The horizontal dashed lines represent the uncoupled hub modes and the slanted dashed lines represent the uncoupled blade modes. As can be seen, at the lower and higher rotor speeds the modes are essentially uncoupled, whereas for the intermediate rotor speeds a considerable amount of coupling is apparent.

When one blade damper is removed the results shown in Figure 3 are obtained. These results were calculated using the Floquet transition matrix method and indicate an instability for rotor speeds between 210 and 305 rpm. The nature of this instability can be

determined from an examination of the frequency plot in Figure 3. At the lower rotor speeds the frequencies for the modes labeled 3 and 6 correspond to the uncoupled blade frequency (in the fixed system) for the blade which has no damper. As the rotor speed is increased the frequency plot for mode 3 indicates coupling with the other modes, and at the higher rotor speeds the mode 3 curve has deviated from the uncoupled curve. At the higher rotor speeds the mode 5 curve is nearer the uncoupled blade frequency curve. This behavior indicates that there is a significant amount of coupling between the unstable mode 3 and the mode labeled 2 which is predominantly a hub mode. The conclusion here is that the indicated instability has the same character as a classical ground resonance instability, and that the instability involves a considerable amount of blade motion.

Figure 4 presents the results of a numerical integration of the equations of motion for a rotor speed of 255 rpm. This rotor speed corresponds to the point of maximum instability in Figure 3. As can be seen from the time histories in Figure 4, blade 1 which has no lag damper is experiencing large excursions. It should also be noted from this figure that determination of whether the system is stable or unstable requires considerable judgment on the part of the analyst. The blade 1 trace appears to be stable, whereas it is impossible to make a definite conclusion relative to the hub traces. This serves to illustrate the fact that time history solutions are less desirable than eigenvalue methods for determining the dynamic stability of systems. The problem with the time history solutions is that one can never be sure that the equations have been integrated over a sufficiently long period for the initial conditions chosen. If the initial conditions are not chosen so as to excite the mode of instability, an extremely long period of integration may be necessary. A further drawback of the numerical integration method is that, in general, it requires much more computing time than does the eigenvalue approach.

Results obtained using the smearing approach are illustrated in Figure 5. Note that although the mode labeled 3 becomes lightly damped, the system remains stable throughout the rotor speed range considered. The smearing technique is thus not recommended for treating the one-blade-damper-inoperative situation since it leads to unconservative results.

Suppose, however, that a designer were using the smearing technique and obtained the results shown in Figure 5. The logical approach to making the system more stable would be to add additional blade lag damping since this parameter is known to be quite effective in eliminating the classical ground resonance. The effect would be that the smearing technique would then indicate a sufficient stability

margin since it is simply an application of the classical analysis. Note, on the other hand, what the correct results, as obtained from the Floquet analysis and shown in Figure 6, would indicate. As can be seen from this figure, the increase of blade lag damping has no effect on the region of instability when one blade damper is inoperative. This result is not too surprising since it was observed from the time history traces of Figure 4 that the blade with no damper is responding more or less independently of the other blades. These results further strengthen the conclusion that the smearing technique should not be used for examining the one-blade-damper-inoperative ground resonance problem.

From the results of Figure 6 it might appear that the Army had imposed an impossible requirement on the helicopter designer. As will be shown, however, it is possible to eliminate the instability through proper selection of the parameters available to the designer. Since it has been shown that increasing the lag damping has no effect on the region of instability with one blade damper inoperative, the only other blade parameter which the designer can vary is the lag spring rate. Coleman and Feingold (1) have shown that, in the absence of lag damping, ground resonance is impossible if the blade lag frequency is greater than the rotor speed. This requirement is extremely conservative, however, if blade lag dampers are present.

Figure 7 illustrates the effect of lag spring rate on the region of instability when one blade damper is inoperative. The assumption was made here that the lag spring and lag damper were independent so that failure of the lag damper did not result in simultaneous failure of the lag spring. Thus the results of Figure 7 were obtained for a rotor in which each of the blades had the same spring rate, but one blade damper was inoperative. As can be seen, if the lag spring rate is made high enough the region of instability can be eliminated. The spring rate required is, on the other hand, much lower than the spring rate necessary to make the blade lag frequency equal to the rotor speed. It is felt, however, that the lag spring rate required is unrealistic and, further, the higher the spring rate the larger will be the vibratory loads transferred to the fuselage. Thus the lag spring does not appear to be the optimum parameter to use in eliminating the instability.

The other parameters available to the designer are the hub parameters. The hub stiffness and damping in the lateral and longitudinal directions may be changed by altering the landing-gear geometry and oleo characteristics. Further, the oleo stiffness and damping are usually adjustable in service so that these are potentially powerful parameters for the designer. Figure 8 shows the effect of lateral hub

damping on the region of instability. This figure illustrates that hub damping can be used effectively in eliminating the instability. This can be attributed to the fact that the only means for the blade with no damper to experience any damping is through the hub coupling terms in the equations of motion. Thus the addition of hub damping tends to stabilize any unstable hub motions while providing damping for the large blade excursions.

The effect of variation of the lateral hub spring rate is shown in Figure 9. These results indicate that, if the stiffness of the hub is reduced, the instability can be eliminated. Again the indication is that the coupling between the rotor and hub is the key to eliminating the instability. Here the lower stiffness hub is allowed to respond more than the higher stiffness hub and thus a greater amount of hub damping can be transferred to the blade through the coupling terms.

As a final result, it was thought to be of interest to determine how much damping could be removed from one blade before encountering an unstable region. The results of these calculations are shown in Figure 10. From Table 1 it may be noted that each blade originally had a blade damper whose rate was 3000 ft-lb-sec/rad, and from the figure it is seen that an unstable region appears when the damping on one blade is reduced to approximately 1000 ft-lb-sec/rad. Thus, for this particular example, approximately two-thirds of the damping may be removed from one blade before an instability results.

CONCLUSIONS

Three main conclusions may be drawn from the results presented. First, the analyst must be careful to examine new user requirements to ascertain that the assumptions of existing analytical tools are not violated. If these assumptions are violated, modifications to the analysis must be made or new analyses must be formulated to handle the new requirements. Modifying the physical problem so that it fits existing analytical methods can lead to erroneous conclusions as evidenced by the smearing technique results.

The most effective means of eliminating the mechanical instabilities which occur with one blade damper inoperative appears to be through appropriate adjustment of the hub effective stiffness and damping characteristics. The approach seems to be to either reduce the hub stiffness to allow more hub response or increase the hub damping. A combination of reduced hub stiffness and increased hub damping will probably provide the most nearly optimum solution.

Finally, the Floquet transition matrix method is an effective analytical tool for dealing with the one-blade-damper-inoperative ground resonance problem. The method provides the stability boundaries directly and thus eliminates the uncertainties associated with time history solutions.

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TABLE 1. PARAMETERS USED IN THE SAMPLE CALCULATIONS

Number of blades	4
Blade mass, m_b	6.5 slugs (94.9 kg)
Blade mass moment, S_b	65.0 slug-ft (289.1 kg-m)
Blade mass moment of inertia, I_b	800.0 slug-ft^2 (1084.7 kg-m^2)
Lag hinge offset, e	1.0 ft (0.3048 m)
Lag spring, k_i	0.0 ft-lb/rad (0.0 m-N/rad)
Lag damper, c_i	3000.0 ft-lb-sec/rad (4067.5 m-N-s/rad)
Hub mass, m_x	550.0 slugs (8026.6 kg)
Hub mass, m_y	225.0 slugs (3283.6 kg)
Hub spring, k_x	85000.0 lb/ft (1240481.8 N/m)
Hub spring, k_y	85000.0 lb/ft (1240481.8 N/m)
Hub damper, c_x	3500.0 lb-sec/ft (51078.7 N-s/m)
Hub damper, c_y	1750.0 lb-sec/ft (25539.3 N-s/m)

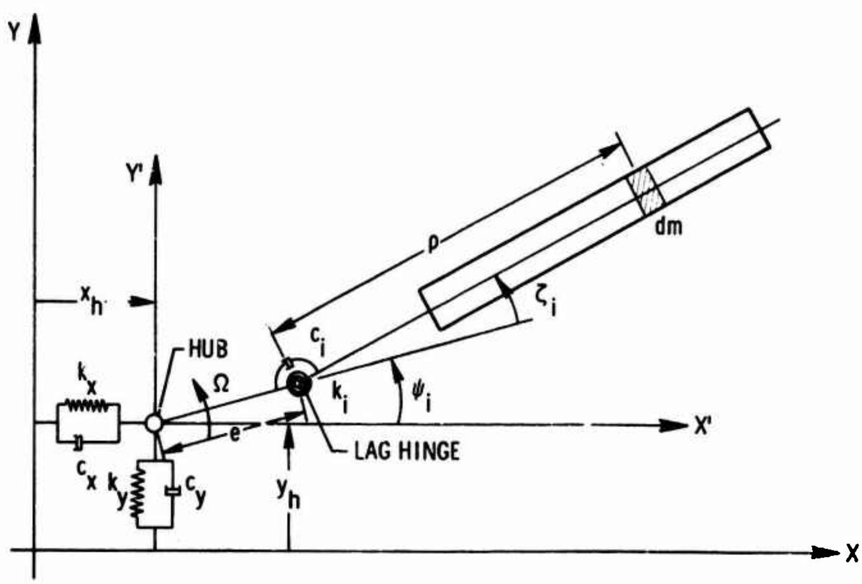

Figure 1. Mathematical representation of the rotor and hub.

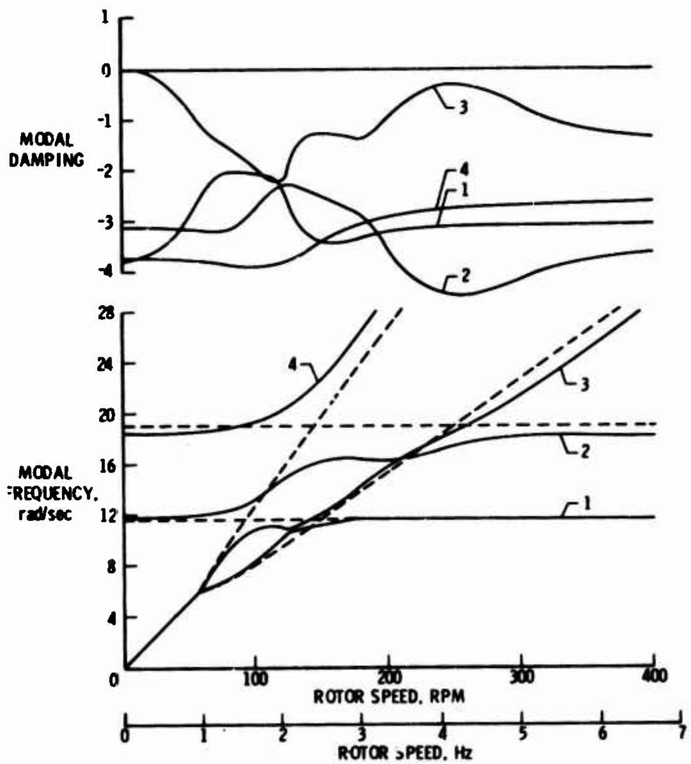

Figure 2. Modal damping and frequencies for nonisotropic hub, all blade dampers working. Frequencies plotted in the fixed system.

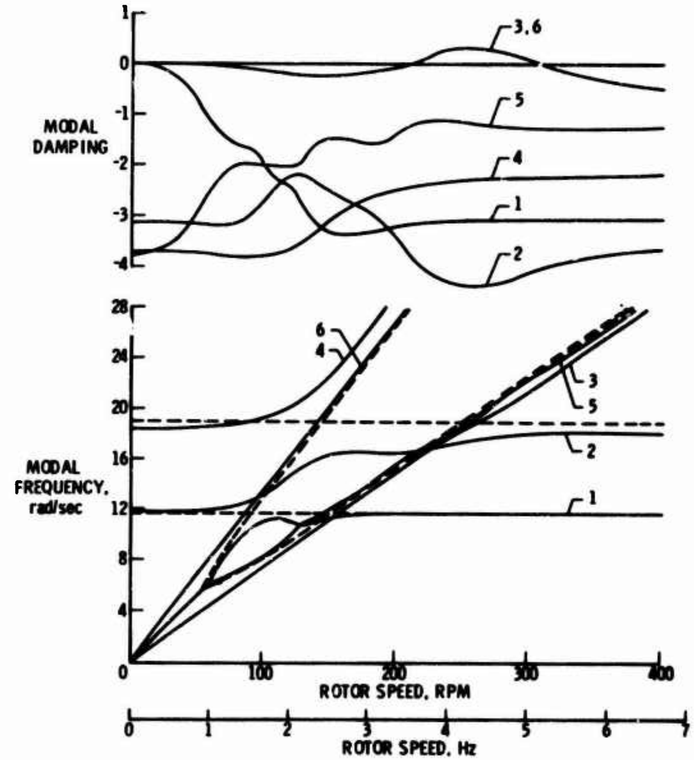

Figure 3. Modal damping and frequencies for nonisotropic hub, one blade damper inoperative. Frequencies plotted in the fixed system.

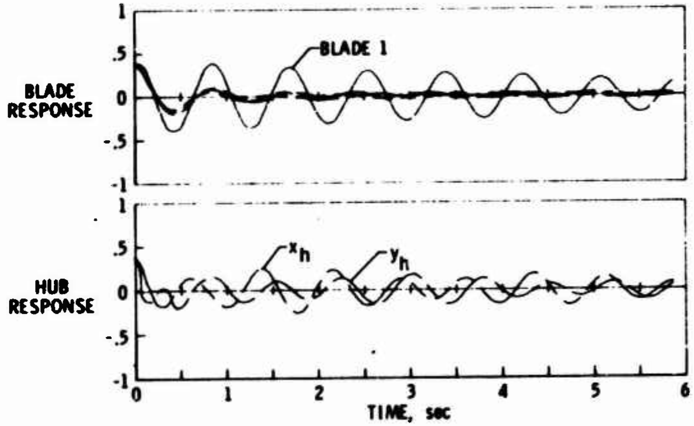

Figure 4. Time history calculations for nonisotropic hub, one blade damper inoperative, $\Omega = 255$ rpm.

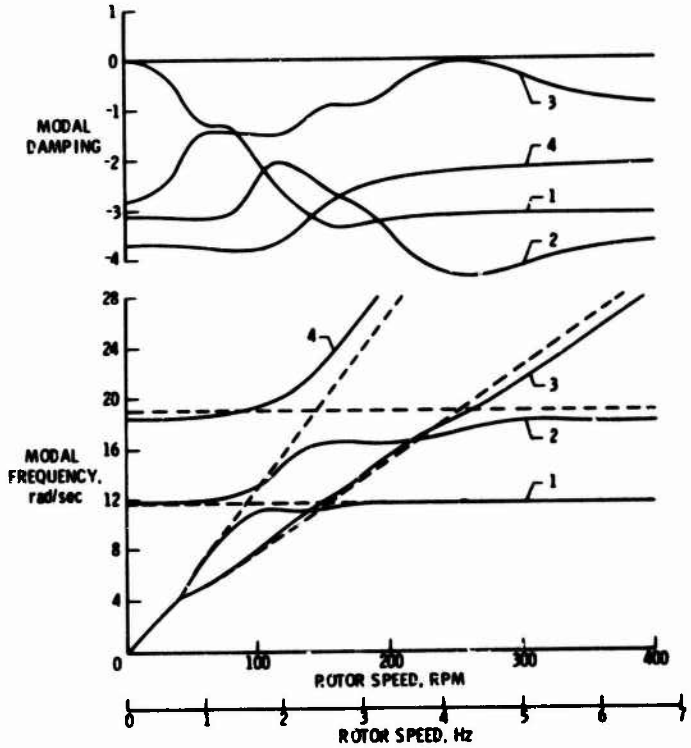

Figure 5. Modal damping and frequencies obtained for nonisotropic hub, one blade damper inoperative, using the smearing technique.

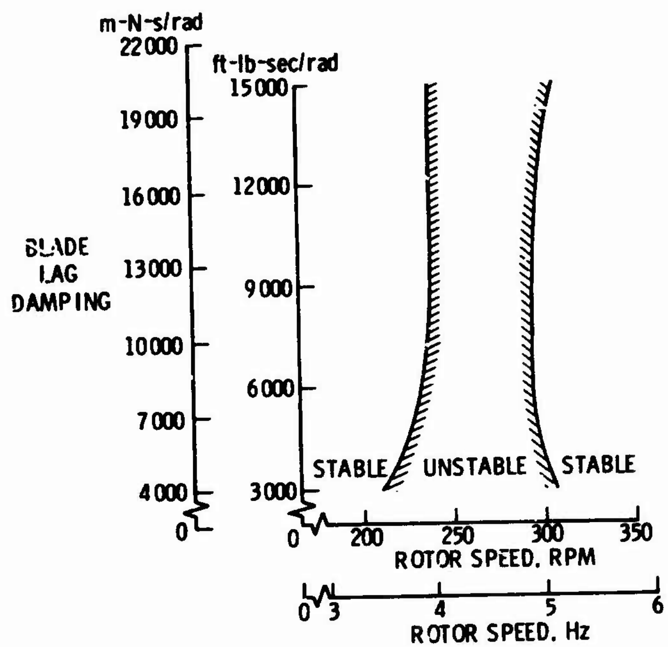

Figure 6. Instability region as a function of blade lag damping for the non-isotropic hub and one blade damper inoperative.

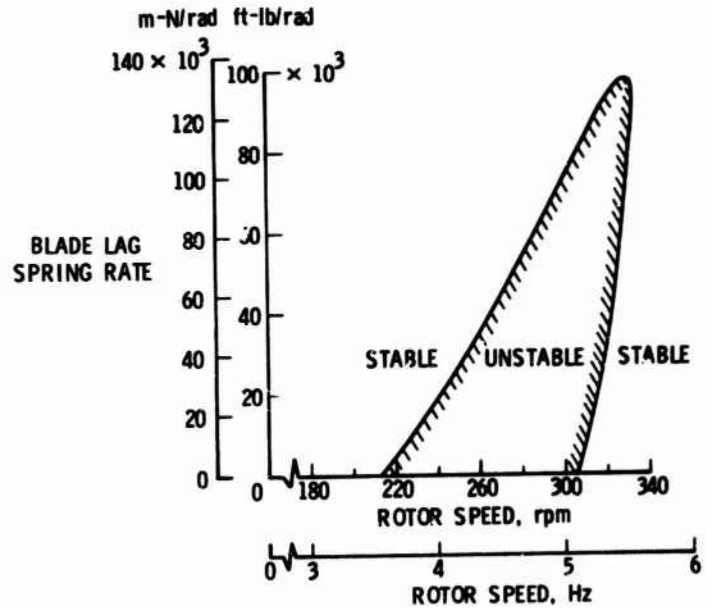

Figure 7. Effect of blade log spring rate on region of instability for one blade damper inoperative.

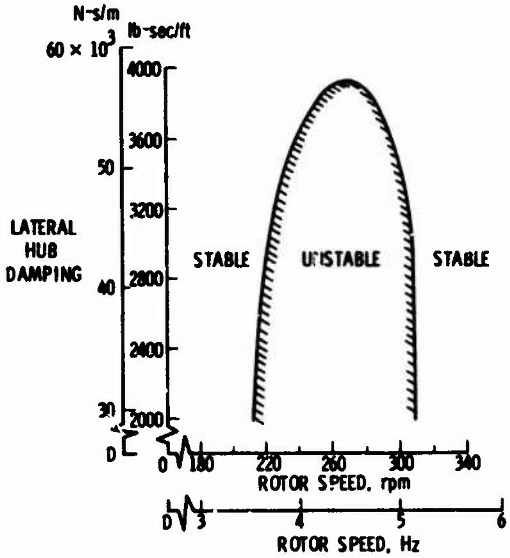

Figure 8. Effect of lateral hub damping on region of instability with one blade damper inoperative.

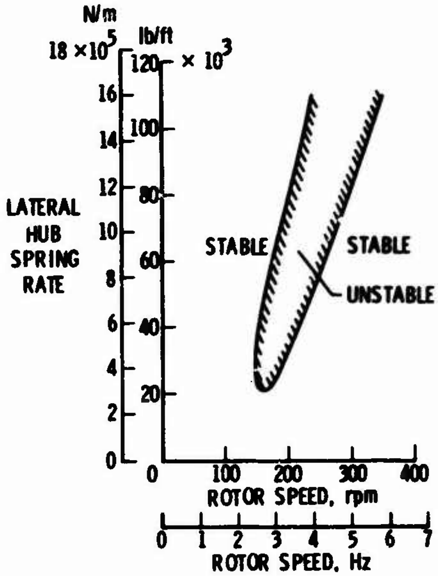

Figure 9. Effect of lateral hub spring rate on region of instability with one blade damper inoperative.

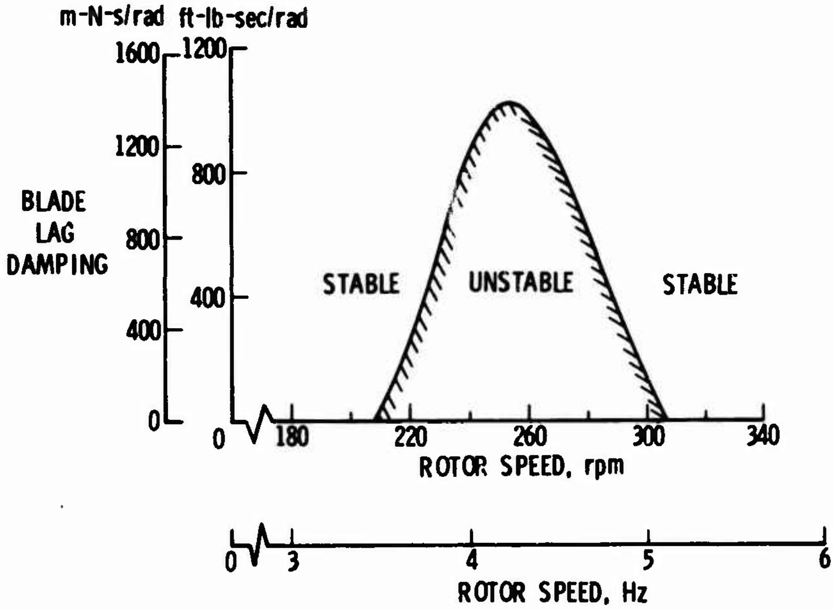

Figure 10. Region of instability resulting from reduced lag damping on one blade.